Sec. 8.4 Trigonometric Equations and Inverse Functions

Solving Trigonometric Functions Graphically:

1. Enter each side of the equation into y_n=.
2. Find the point of intersection(s).

Ex: Given that the rabbit population can be found by $R = -5000\cos\left(\frac{\pi}{6}t\right) + 10000$, find the time when the population reaches 12,000 rabbits.

Solving Trigonometric Functions Algebraically:

Ex: Now find the solution to the rabbit situation $R = -5000\cos\left(\frac{\pi}{6}t\right) + 10000$ algebraically.

$12{,}000 = -5000\cos\left(\frac{\pi}{6}t\right) + 10{,}000$

$2000 = -5000\cos\left(\frac{\pi}{6}t\right)$

$-\frac{2}{5} = \cos\left(\frac{\pi}{6}t\right)$

$\frac{\pi}{6}t = \cos^{-1}\left(-\frac{2}{5}\right)$

$\frac{\pi}{6}t = 1.9823$

$t = 3.786$ months

$\frac{\pi}{6}t = 2\pi - 1.9823$

$\frac{\pi}{6}t = 4.3009$

$t = 8.214$ months

The **inverse cosine** function, also called the arccosine function, is written $\cos^{-1} y$ or arccos y.

We define $\cos^{-1} y$ as the angle between 0 and π whose cosine is y. More formally, we say that

$$\mathbf{t = \cos^{-1} y \text{ provided that } y = \cos t \text{ and } 0 \le t \le \pi.}$$

Note that for the inverse cosine function:

- the domain is $-1 \le y \le 1$
- the range is $0 \le t \le \pi$.

Ex. Evaluate without a calculator:

(a) $\cos^{-1} 0$

$\frac{\pi}{2}$

(The angle between 0 and π that has a cosine of 0.)

(b) arccos(1)

0

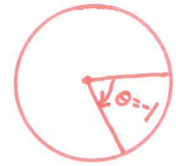
(The angle between 0 and π that has a cosine of 1)

(c) $\cos^{-1}(-1)$

π

(The angle between 0 and π that has a cosine of -1)

(d) $(\cos(-1))^{-1}$



$\cos(-1) = .540$

{cosine of $-57.3°$}

$(.540)^{-1} = \frac{1}{.540}$

1.851

The **inverse sine function**, also called the arcsine function, is denoted by $\sin^{-1} y$ or arcsin y. We define:

$$\mathbf{t = \sin^{-1} y \text{ provided that } y = \sin t \text{ and } -\pi/2 \le t \le \pi/2.}$$

The inverse sine has domain $-1 \le y \le 1$ and range $-\pi/2 \le t \le \pi/2$.

The **inverse tangent function**, also called the arctangent function, is denoted by $\tan^{-1} y$ or arctan y. We define:

$$\mathbf{t = \tan^{-1} y \text{ provided that } y = \tan t \text{ and } -\pi/2 < t < \pi/2.}$$

The inverse tangent has domain $-\infty < y < \infty$ and range $-\pi/2 < t < \pi/2$.

Ex: Evaluate:

(a) $\sin^{-1}(1)$

$\frac{\pi}{2}$

(Angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ that has a sine of 1)

(b) arcsin(−1)

$-\frac{\pi}{2}$

(Angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ that has a sine of −1)

(c) $\tan^{-1}(0)$

0

(Angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ that has a tan of 0 – when sin = 0

(d) arctan(1)

$\frac{\pi}{4}$

(Angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ that has a tan of 1 -- when sin = cos)

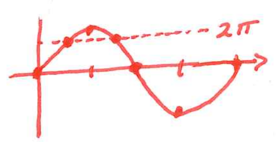
Ex: A. Solve $\sin\theta = 0\ 9063$ for $0 \le \theta \le 2\pi$. Then write all solutions for the equation.

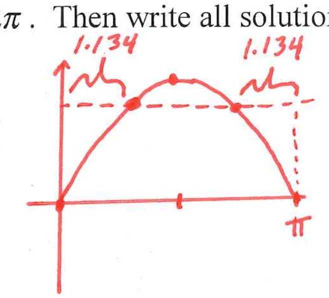
$\theta = \sin^{-1}(.9063)$

$\theta = \boxed{1.134}$

{Between $-\frac{\pi}{2}$ & $\frac{\pi}{2}$}

$\theta_2 = \pi - 1.134 = \boxed{2.007}$





$$\boxed{\theta = \begin{cases} 1.134 + 2k\pi \\ 2.007 + 2k\pi \end{cases}}$$

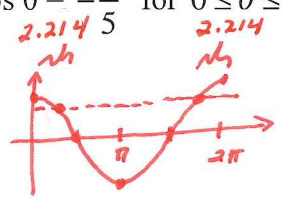
B. Solve $\cos\theta = -\frac{3}{5}$ for $0 \le \theta \le 2\pi$. Then write all solutions for the equation.

$\theta = \cos^{-1}(-\frac{3}{5})$

$\boxed{\theta = 2.214}$

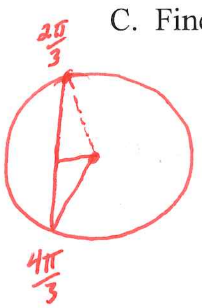
$\theta_2 = 2\pi - 2.214$

$\boxed{\theta_2 = 4.069}$



$$\boxed{\theta = \begin{cases} 2.214 + 2k\pi \\ 4.069 + 2k\pi \end{cases}}$$

C. Find all exact values to $\cos\theta = -\frac{1}{2}$



$$\theta = \begin{cases} \frac{2\pi}{3} + 2k\pi \\ \frac{4\pi}{3} + 2k\pi \end{cases}$$

For an angle θ corresponding to the point P on the unit circle, the **reference angle** of θ is the angle between the line joining P to the origin and the nearest part of the *x*-axis. A reference angle is always between 0° and 90°; that is, between 0 and $\pi/2$.

HW: pg 347 – 349 #2,4,6,10,12,16,22,26,32,34,35,43